FAMILY GUIDE

7th Grade Math

Unit 1: Proportional Relationships

OVERVIEW

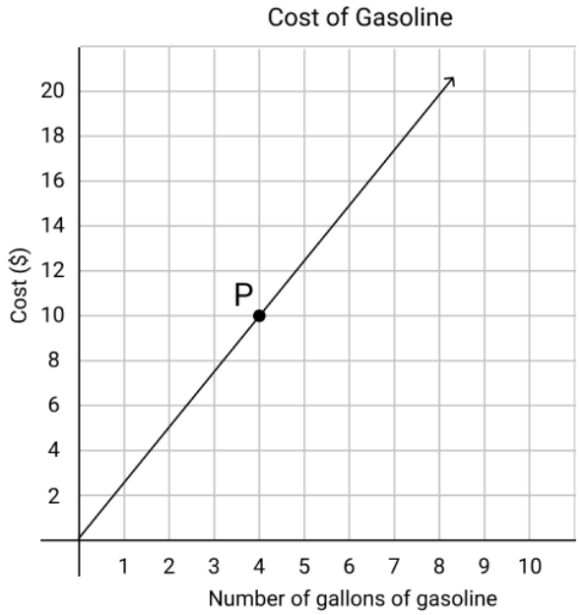
In this unit, students deepen their understanding of ratios, which they studied in sixth grade, and begin to learn about proportional relationships. They understand proportional relationships as collections of equivalent ratios.

First, they look at how proportional relationships are represented in tables, equations, and graphs. As they study each representation, students begin to understand what proportionality means, and how it can be visible in different ways. Knowing one representation provides the information needed for students to represent the relationship in a different way. (See the table below for examples.)

Students then spend time comparing examples of proportional and non-proportional relationships. They'll investigate why situations like a unit price per pound will lead to a proportional relationship between cost and number of pounds; however, a situation like a brother who is 4 years older than his sister will not lead to a proportional relationship between their ages.

Finally, students solve real-world ratio and rate problems with fractions, including problems with constant speed, unit prices, price increases and decreases, and fees. Later in the year, students will return to proportional reasoning when they study percent problems and scale drawings.

<table>
<tr><th>Model</th><th>Example</th></tr>
<tr><td>Setting up and solving a proportion</td><td>A group of 4 students buy movie tickets for $24. At this rate, how much would 20 students pay for the movie?
$\frac{4 \text{ students}}{\$24} = \frac{20 \text{ students}}{\$x}$
$4x = 24(20)$
$x = \$120$</td></tr>
<tr><td>Table of equivalent ratios</td><td>The table below shows some weights of rice, in pounds, and their corresponding costs, in dollars.
<table>
<tr><th>Rice (lbs.)</th><th>Cost ($)</th></tr>
<tr><td>2</td><td>11</td></tr>
<tr><td>10</td><td>55</td></tr>
<tr><td>13</td><td>?</td></tr>
<tr><td>?</td><td>88</td></tr>
</table></td></tr>
<tr><td>Equation</td><td>The equation $y = 8.75x$ represents the cost in dollars, y, to purchase x pounds</td></tr>
</table>

	of turkey meat at a deli.
Graph	The graph below shows the relationship between the cost of gas and the number of gallons of gas purchased at a gas station. 

Check out the Student Self-Assessment to read about the unit's standards in student-friendly language.

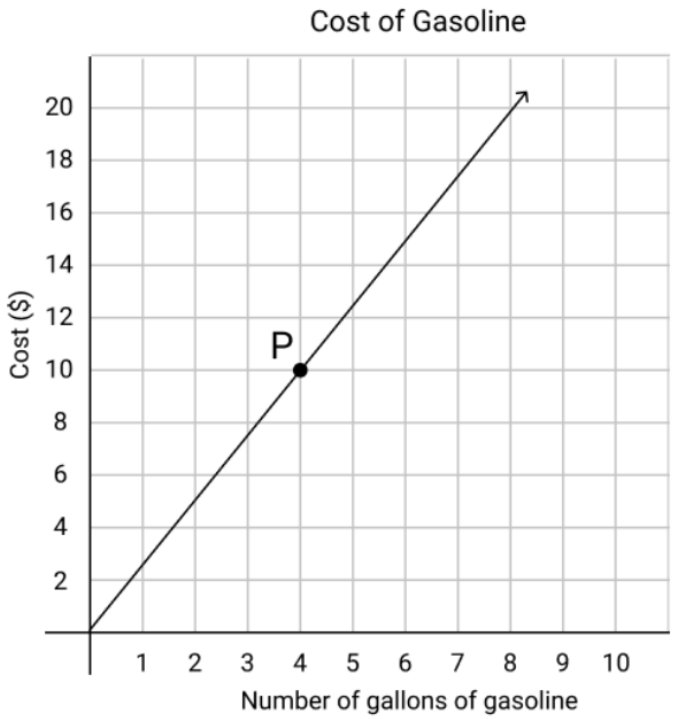
To access this document online, visit FishtankLearning.org and navigate to this unit's page.

EXAMPLE PROBLEMS FROM UNIT

Here are some example problems and student responses from the unit that highlight some of the key concepts students will study:

From Lesson 6, Target Task: Understanding proportional relationships in graphs

The graph below shows the relationship between the cost of gasoline and the number of gallons of gasoline purchased at a gas station.



a. Is the cost of gasoline proportional to the amount of gasoline purchased? Explain how you know using the graph.

b. How many gallons of gasoline can you purchase with $15?

c. How much can you expect to pay for 3 gallons of gasoline?

d. What are the coordinates of point P, shown in the graph? Describe what point P means in context of the situation.

Student Response

a. Yes, the cost and number of gallons purchased are proportional. The graph is a straight line through the origin.

b. 6 gallons

c. $7.50

d. (4,10) you will pay $10 for 4 gallons of gas.

From Lesson 8, Target Task: Comparing proportional and nonproportional situations

Two ice cream shops are located across the street from one another on a busy street.

- At Max's Ice Cream shop, you can buy a sundae with unlimited toppings for $5 per sundae.
- At Mary's Ice Cream shop, you can buy a sundae by paying $3.50 for the ice cream and then $0.75 for each topping.

At which ice cream shop is the cost of the sundaes (including the toppings), proportional to the number of sundaes purchased? Justify your answer with tables, graphs, or an explanation.

Student Response

At Max's Ice Cream Shop, the cost of sundaes is proportional to the number of sundaes purchased. There is <u>not</u> a proportional relationship between these two quantities at Mary's Ice Cream Shop.

At Max's, every sundae costs $5, which can be represented by the equation $c = 5s$, where c is the total cost and s is the number of sundaes.

At Mary's, the cost depends on the number of toppings ordered. One sundae could cost $3.50 with no toppings, or $4.25 with one topping. There is no constant of proportionality.

From Lesson 14, Target Task: Using unit rate to solve problems

(In 7th grade, the focus is on working with unit rates that arise from working with fractions.)

At a candy store, you can buy pre-filled bags of candy that weigh $2\frac{1}{4}$ pounds for $\$8.10$. At this same rate, how much would a 4 pound bag of candy cost?

Student Response

$\frac{8.10}{2\frac{1}{4}} = 3.60$ or $3.60 per pound

So a 4 pound bag of candy would cost 4×3.60 or $14.40.

From Lesson 15, Target Task: Using a proportion as a strategy to solve problems

The ratio of boys to girls at a soccer camp is 3:5. If there are 51 boys at the camp, then how many of the campers are girls?

Solve using a proportion, and then choose one other method to check your answer.

Student Response

$\frac{\text{boys}}{\text{girls}}: \frac{3}{5} = \frac{51}{x}$ $\quad \frac{3x}{3} = \frac{5(51)}{3}$

$x = 85$ girls

check using a table:

Boys	Girls
3	5
51	?

×17 ×17

$5 \times 17 = 85$ ✓

CONNECTIONS AT HOME

Encourage your 7th grader to find math problems in everyday life by asking, "what math question can we ask or answer in this scenario?" This helps students see connections between math and everyday moments. Plus, having students ask the math questions (instead of being asked) can be empowering. Since there are infinite questions that can be asked and answered about the same context, this is an endless source of conversation about math!

Here are some ideas of how you can make connections to everyday life using ideas from this unit:

- Notice and talk about different ratio relationships seen everyday. For example, a great place to start might be noticing the ratios of items in their wardrobe. A snapshot of this could be thinking about ratios of the items they would pack for a weekend trip.
- As students might help in the kitchen, you can talk about ratios that are used to make meals for the family. Think about what ratios would be involved to scale a recipe down for an individual or to scale a recipe up when there are additional guests.
- Shopping is a great connection to unit rates. Encourage students to help uncover the best deals and explain how they came to that conclusion. On price labels, look to see if the cost per unit is included and ask how this information helps determine the best deal. Ask students why they think larger quantities of items have a lower unit rate than smaller quantities (such as the cost per ounce of milk bought as a gallon vs. the cost of milk per ounce bought as a quart).

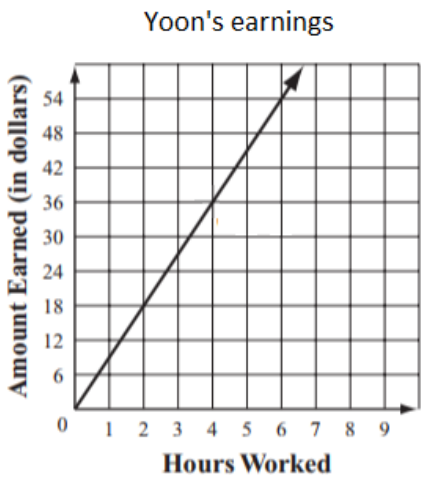
Talking about math

Talking math with your 7th grader is a great way to help them build their skills and confidence. To do this with your student, you don't have to be an expert! Let them tell you about what they know. If there are topics you both have additional questions about, problem solve and do research together. Don't give up if you don't understand. Instead, help your 7th grader make a plan to get help from their teacher–this models mathematical perseverance and builds an important skill in knowing when and who to ask for help. It also shows your 7th grader that learning is a lifelong pursuit!

Here are some questions you can use with your 7th grader to talk about their math work:

- Tell me about a problem you enjoyed solving in math class.
- Tell me about a tricky problem this week and how you persevered to solve it.
- Tell me about what you tried and why you think it didn't work.
- What strategy are you most excited about in this unit?
- Did you learn a new strategy from someone else in your class? Did you teach someone else about a strategy that you use?
- Review a graded assignment together. How might you solve this problem differently? Tell me what you know about this problem? How does that help you answer it?

VOCABULARY

Word	Definition	Example
commission	An amount of money, usually determined by a percentage of a value, that is paid in a transaction.	Employees at the furniture store earn a 3% commission on the price of each piece of furniture they sell. For example, if an employee sells a couch for $1200, then the employee earns a commission of 3% of $1200, or $36.
constant of proportionality	In a proportional relationship, the constant of proportionality is the constant multiplier between the two quantities. In the equation $y = kx$, it is represented by the variable k. In a graph of a proportional relationship, the constant of proportionality is represented by the variable r in the coordinate point $(1, r)$.	The graph below shows Yoon's earnings over time. The constant of proportionality is 9, as seen in the coordinate point (1, 9), meaning Yoon earns $9 for each hour she works. 
dependent variable	The dependent variable represents the output in an equation. The value of a dependent variable depends on the value of another.	The equation $d = 3.5t$ represents the distance, d, in miles that Kyle walks over time, t, in hours. The variable d is the dependent variable because the distance that Kyle walks is dependent on the amount of time he walks for.

Word	Definition	Example
equivalent ratio	Two ratios are equivalent if there is a nonzero number that can be multiplied by both quantities in one ratio to equal the corresponding quantities in the second ratio. The ratio of $A : B$ is equivalent to $c \times A : c \times B$ for a nonzero number c.	Ratios $2 : 5$ and $6 : 15$ are equivalent because 2×3 is 6 and 5×3 is 15.
independent variable	The independent variable represents the input in an equation. The value of an independent variable does not depend on any other value.	The equation $d = 3.5t$ represents the distance, d, in miles that Kyle walks over time, t, in hours. The variable d is the independent variable because it does not depend on the distance Kyle traveled.
part to part ratio	A ratio that represents the relationship between two different parts.	In a bowl of fruit, there are 6 apples and 2 bananas. The ratio of apples to bananas is 3:1.
part to whole ratio	A ratio that represents the relationship between a part and the whole.	In a bowl of fruit, there are 6 apples and 2 bananas. The ratio of apples to fruit is 3:4.
proportion	A proportion is an equation that states two ratios are equal to each other. (For the definition of proportion in a statistical sense, see sample proportion and population proportion.)	$\frac{3}{5} = \frac{78}{130}$ $\frac{2}{9} = \frac{x}{108}$

Word	Definition	Example
proportional relationship	A relationship between two quantities where the values for one of the quantities are all multiplied by the same number to get the values for the other quantity; there is a multiplicative relationship between the two quantities. This multiplier is called the constant of proportionality. A proportional relationship between two quantities is a collection of equivalent ratios of those quantities.	If x is proportional to y such that y is four times the value of x, then the equation $y = 4x$ can be written to represent the proportional relationship. The constant of proportionality is 4. x / y 1 / 4 2 / 8 5 / 20 8 / 32
rate	A rate is associated with a ratio, such as $a : b$, and is $\frac{a}{b}$ units of the first quantity per 1 unit of the second quantity.	3 pounds of grapes cost \$6, which is a rate of \$2 per pound.
ratio	A set of numbers that associates two or more quantities.	The ratio of students to teachers on the field trip was 8:1.
unit rate	The number of units of the first quantity in a ratio for every 1 unit of the second quantity. The unit rate is the numerical part of a rate.	For the rate of 55 miles per hour, the unit rate is 55, meaning there are 55 miles traveled per every 1 hour.